MATH 120A Prep: Functions

Facts to Know:

Function Properties: Consider a function $f : X \rightarrow Y$.

- Injective/One-to-one -

- Surjective/Onto -

- Bijective -

Examples:

1. (a) Determine whether the exponential map $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^x$ is injective and/or surjective.

   (b) What changes if we consider this as a function $f : \mathbb{R} \rightarrow \mathbb{R}^+$ where $\mathbb{R}^+ = \{r \in \mathbb{R} : r > 0\}$?

2. Is the map $g : \mathbb{R}^2 \to \mathbb{R}$ where $g(x, y) = x^2 - y^2$ injective? Is it surjective?

3. Let S be the set $\{(x, y) \in \mathbb{R}^2 : x \neq y\}$. Show the map $h : S \to \mathbb{R}^2$ defined by $h(x, y) = (x - y, x^2 - y^2)$ is injective but not surjective.